Randomness and Determinism: Why Are the Planetary Orbits Elliptical?

Oscar Sheynin*

Abstract

Kepler attempted to prove Divine design in the system of the world but actually had to attribute the eccentricities of the planetary orbits to randomness. Kant and even Laplace supported Kepler's conclusion although Newton had proved that the eccentricities depended on the velocities of planetary motion. However, the velocities themselves are random; the system of the world does not exclude randomness.

Key words: eccentricities of planetary orbits; randomness in nature; system of the world.

1. Randomness: General Information

Aristotle[1] and other early scientists and philosophers attempted to define, or at least to throw light upon randomness. His examples of random events are a sudden meeting of two acquaintances (*Phys*. 196b30) and a sudden unearthing of a buried treasure (*Metaphys*. 1025a). In both cases the event occurred without being aimed at. Many ancient authors had been repeating the first example and Cournot (1843, § 40) revived it. It is usually interpreted as an intersection of two independent chains of events. Both examples illustrate one of **Poincaré's** explanations (interpretations) of randomness initially contained in his popular book of 1907 and then in his treatise: if equilibrium is unstable,

A very small cause which escapes us determines a considerable effect […] *and we say that that effect is due to chance*[2].

His deliberations (also see below) heralded the beginning of the modern period of studying randomness.

Not less important is Aristotle's explanation of the appearance of monsters (*Phys*. 199b1; *De generatione anim*. 767b5) as mistakes "in the operation of nature"; he also says that the first

Departure from the type is that the offspring should become female instead of male; […] *as it is possible for the male* [for the father] *sometimes not to prevail over the female* [the mother] […].

Given a large number of births, regularities of mass random events will, however, certainly reveal themselves.

Aristotle did not connect such events with randomness; moreover, he (*De Caelo* 283b1 and in other places) stated that "the products of chance and fortune are opposed to what is, or comes to be, always or usually".

* Wilhelmstr. 130, 10963 Berlin, Germany. e-mail: sheynin@rambler.ru or oscar.sheynin@googlemail.com www.sheynin.de

Nevertheless, we are fully justified in calling them random since they are apparently covered by Poincaré's definition. Therefore, not only *lack of purpose* or law, but also *corruption of,* or *deviation from laws of nature* means randomness, and this idea can be traced at least until **Lamarck** who stated that the deviations from the divine lay-out of the tree of animal life had been occasioned by a "cause accidentelle" (Lamarck 1815, p. 133).

There also, on p. 173, he indicated that the spontaneous generation of organisms was caused by a *très-irrégulière* force. He did not mention randomness, but, when considering the state of the atmosphere, Lamarck stated that it was disturbed by two kinds of causes, including *variables, inconstantes et irrégulières* (Lamarck an 8, 1800, p. 76). Again, no mention of randomness, but then he denied it: no "part of nature" disobeys "invariable laws", therefore "that, which is called chance", does not exist (Lamarck 1810 – 1814/1959, p. 632)[3]. In the first and third case he understood randomness as corruption of the laws of nature; and in his other pronouncements, as lack of such laws.

Spontaneous generation was definitively disproved by Louis Pasteur, but I stress that until then it was apparently always considered random. Witness indeed Harvey:

> *Creatures that arise spontaneously are called automatic* […] *because they have their origin from accident, the spontaneous act of nature* (Harvey 1651/1952, p. 338).

Harvey did not say anything about the essence of accidents, but it seems that he thought them aimless, identified them with lack of law.

In his main contribution to probability, the celebrated *Doctrine of Chances*, De Moivre considered as its main achievement the establishment of "certain rules for estimating how far some sort of Events may rather be owing to Design than Chance" (De Moivre 1756, p. 329). This is a quotation from the reprint of his Dedication of the first edition of the *Doctrine of Chances* to Newton. De Moivre also stated there that he should think himself

> *Very happy if having given* […] *a method of calculating the Effects of Chance* […] *and thereby fixing certain Rules, for estimating how far some sorts of Events may rather be owing to Design than Chance, I could* […] *excite in others a desire* […] *of* […] *learning from your* [Newton's] *Philosophy how to collect* […] *the Evidences of exquisite Wisdom and Design, which appear in the Phenomena of Nature* […].

De Moivre did not define chance, but it seems to follow that if design (aim of nature according to Aristotle) exists, then chance is its corruption; true, design is lacking in games of chance (which he studied), and its corruption is out of question; there, it was lack of any law.

I would say that all this testifies that for De Moivre the main goal of the emerging theory of probability was to study the deviations from the Divine laws of nature. In 1733, his derivation of the normal law of distribution was occasioned by a study of the sex ratio at birth. For him, the initial binomial distribution of those births was a designed deterministic law of nature, the first statistical regularity of nature (with its

parameter only approximately known) and only the actual deviations from it were random. See the final version of that derivation (De Moivre 1756, pp. 252 – 253).

I return to **Poincaré**, to his statement initially appearing in another of his popular books in 1908: he attributed accidental errors of observation to chance since

Their causes are too complicated and too numerous. Here again we only have small causes each of them [now, contrary to his previous definition,] *only producing a small effect; it is because of their combination and their number that their effect becomes formidable*[4].

It would have also been possible to cite coin tosses or variations between individuals of a given species.

Poincaré also formulated a dialectical statement about determinism and randomness much broader than the one following from "deviation from laws of nature", it legitimizes randomness and indirectly defines it but did not say anything about regularities of mass random events:

In no field [of science] *do exact laws decide everything, they only trace the boundaries within which randomness is permitted to move. According to this understanding, the word randomness has a precise and objective meaning*[5].

Recall De Moivre's binomial law of the sex ratio at birth.

Exact laws tolerate randomness if they do not allow for some conditions. Indeed, here is **Newton**:

Blind fate could never make all the planets move one and the same way in orbs concentrick, some inconsiderable irregularities excepted, which may have risen from the mutual actions of comets and planets upon one another, and which will be apt to increase, till this system must be allowed the effect of choice. (Newton 1704/1931, Query 31.)

Perturbations have appeared here just as errors of observations did in Poincaré's reasoning. I will now mention **Laplace** who stated that the arrangement of printed letters in the word *Constantinople* "is not due to chance"; all arrangements are equally unlikely, but that word has a meaning and it is "incomparably more probable" that someone had written it on purpose (Laplace 1814/1995, p. 9, my paraphrase). He equated randomness with lack of purpose.

True, Laplace (1814/1995, p. 2) stated that, for a mind, able to "comprehend" all the natural forces, and to "submit these data to analysis", there would exist no randomness "and the future, like the past, would be open" to it. My example: the outcome of a coin toss will then be predicted, cf. Poincaré's statement (above) about errors of observation. Nowadays, this opinion cannot be upheld since the recently discovered phenomenon of chaos or, I would say, chaotic processes greatly restricted our capability of forecasting[6]. However, other remarks are also in order. Such a mind does not exist. In addition, there are unstable movements, sensitive to small changes of initial conditions. And I also note that

already previous scholars, for example, Maupertuis (1756, p. 300) and Boscovich (1966, §385), kept to the "Laplacean determinism". Both mentioned calculations of past and future ("to infinity on either side", as Boscovich maintained) but both disclaimed any such possibility.

The main pertinent point is, however, that Laplace had actually recognized randomness. Without applying stochastic methods he would have not been engaged in studying and furthering the theory of probability, and neither would have he been able to achieve brilliant success in astronomy. Here is an example (regrettably the only direct confirmation of the above):

That [lunar] *inequality, although indicated by observations, was neglected by most astronomers because, as it seemed, it did not follow from the theory of universal attraction. Nevertheless, subjecting* [the probability of] *its existence to the Calculus of Probabilities, I determined that its probability was very high, and considered myself obliged to study its cause*[7].

In my context, there seems to have been only one scholar between Aristotle and Kepler, to whom I turn in § 2, and that was Thomas Aquinas (Sheynin 1974, § 2.4). His general goal was to unite faith and reason and to adapt pagan Aristotle to Christianity. He repeated the Philosopher's thoughts and mentioned "some hindering cause" (some corruption of law) bringing about the "production of females".

2. Kepler

Kepler only formally denied randomness:

What is, however, randomness? Indeed, the most disgusting idol, nothing but an insult to God, Sovereign and Almighty, as well as to the most perfect world that He created [8].

In astrology, Kepler considered himself the founder of its scientific direction, of studies of the qualitative correlation between heavenly forces and events occurring on the Earth. Leaving aside his predecessors (for example, Ptolemy and Tycho Brahe), I quote his typical statement:

An astrologer who only sees the sky but […] *does not know anything about intermediate causes can only forecast probably* […] *which means a bit better than not at all*[9].

Probably is not definite enough, but the main point is that Kepler actually recognized randomness as corruption of law.

I (Sheynin 1974, § 7) treated Kepler's astrology in much more detail, but now I turn to astronomy, and namely to the problem of eccentricities of the planetary orbits. At first, Kepler understood eccentricity as the preordained eccentric position of the Sun as measured from the centre of the circular orbit of a given planet. He then changed his (actually, ancient) definition and stated that eccentricity depended on the combination of external forces, see below.

Kepler (1596/1963) first encountered those eccentricities when attempting to construct a model of the solar system by inserting the five regular solids between the spheres of the then six known planets: they, the eccentricities, and, for that matter, unequal one to another, much worried him: "The causes of the eccentricities are not yet studied, and neither are their differences" (Die Ursache der Excentrizitäten wie auch ihrer Unterschiede noch nicht erforscht ist; Chapter 18, p. 111).

In Chapter 17, p. 108, he formulated the problem for those interested: To discover these causes by issuing from the regular solids. God, he added, did not assign the eccentricities accidentally. In the second edition of that contribution Kepler provided Notes to almost each chapter, and we find there that that problem was not solved [by his predecessors] (p. 117) but that he had investigated it, "and look, I have [he had] revealed the main (vorzüglichsten) causes" (p. 118 with a reference to Book 5 of his *Harmony* (1619)).

Here is the title of one of the chapters of that contribution:

The origin of the eccentricities of the individual planets [is] *in the arranging of the harmonies between their motions* (Kepler 1619/1997, title of Chapter 9 of Book 5 on p. 451).

On that same page he explained that God had combined the planetary motions with the five regular solids and thus created the only most perfect prototype of the heaven.

Again in the same chapter, in Proposition 5, on p. 454, he indirectly mentioned in this connection his second law of planetary motion; for that matter, he could have referred to it in his *Epitome* (1618 – 1621). Even admitting his theory of solids, which definitively fell down after the discovery of the seventh planet (Uranus), we see, however, that Kepler did not explain the values of those eccentricities. In other words, randomness persisted in spite of his efforts, and its cause was left obscure.

In his main work, Kepler indicated that

Examples of natural things, and the kinship of celestial things for these terrestrial ones […], *cry out that* […] *the variables, if any (such as, in the motion of the planets, the varying distance from the sun, or the eccentricity* [which explains why do the distances vary] *arise from the concurrence of extrinsic causes* (Kepler 1609/1992, Chapter 38, pp. 404 – 405).

On the same page 405 he illustrated his opinion by obstacles which prevent rivers from descending "towards the centre of the earth", and finally, on the next page, he concluded that "other causes are conjoined with the motive power from the sun" [affect their motion], cf. deviation from laws of nature (§ 1).

Kepler voiced his main statement in a later contribution:

If the celestial movements were the work of mind, as the ancients believed, then the conclusion that the routes of the planets are perfectly circular would be plausible. […] *But the celestial movements are* […] *the work of* […] *nature* […] *and this is not proved by anything more validly*

than by observation of the astronomers, who […] *find that the elliptical figure of revolution is left in the real and very true movement of the planet.* […] *Because in addition to mind there was then need of natural and animal faculties* [which] *followed their own bent* […] [and] *did many things from material necessity. So it is not surprising if those faculties, which are mingled together, could not attain perfection completely. The ancients themselves admit that the routes of the planets are eccentric, which seems to be a much greater deformity than the ellipse.* (Kepler 1618 – 1621, 1620/1952, Book 4, pt. 3, § 1, p. 932).

Or, more subtly: attempts to obey laws of nature which are, however, too complicated to follow, involve those same deviations.

3. Kant and Laplace

I do not know if or to what extent had Kant borrowed from Kepler, but in any case he held to external influences, – again to deviations or complications preventing obedience to laws of nature:

The multitude of circumstances that participate in creating each natural situation, does not allow the preordained regularity to occur[10].

Why are their [the planets'] *paths not perfectly circular? Is it not seen clearly enough, that the cause that established the paths of celestial bodies* […] *had been unable to achieve completely its goal?* […] *Do we not perceive here the usual method of nature, the invariable deflection of events from the preordained aim by various additional causes*?[11]

And now I turn to Laplace:

Had the Solar system been formed perfectly orderly, the orbits of the bodies composing it would have been circles whose planes coincide with the plane of the Solar equator. We can perceive however that the countless variations that should have existed in the temperatures and densities of the diverse parts of these grand masses gave rise to the eccentricities of their orbits and the deviations of their movement from the plane of that equator (Laplace 1884, Note 7, p. 504)[12].

The causes mentioned by Laplace could have hardly be called external, but one of the main relevant explanations of randomness, deviation from the laws of nature, persisted.

4. Newton

Newton theoretically proved that the Keplerian laws of planetary motion resulted from his law of universal gravitation. In my context, it is necessary to stress: it is generally known that he also established that the eccentricity of the orbit of a given planet was determined by the planet's initial velocity. For some greater values of that velocity the orbit will become parabolic (with its eccentricity ε equal to unity, not less than unity as in the case of ellipses), for other still greater values, hyperbolic (with $\varepsilon > 1$). And for a certain value of that velocity an elliptic orbit will become circular. And it is difficult to imagine that such changes do not occur gradually, that, consequently, the eccentricity does not vary continuously with the velocity.

All these findings, as Newton proved, persisted for planets (not material points) having a regularly variable density. I believe that irregular variations of densities (but hardly temperatures) peculiar to a given planet (Laplace) could have only somewhat corrupted the eccentricity caused by its initial velocity and in any case Laplace did not provide any calculations.

5. Discussion

In spite of his formal denial of randomness, Kepler had at least sometimes actually acknowledged it. Whatever he could have thought, his laws did not explain the values of the eccentricities. But it really seems that Laplace (and Kant) were mistaken (Kepler was obviously ignorant of the law of universal gravitation). I am not sure that Kant had studied Newton attentively enough, but Laplace certainly did, and I am unable to explain his statement.

Witness finally Fourier's comment on Laplace's *Exposition*: it "is an ingenious epitome of the principal discoveries" (Fourier 1829, p. 379). And on the same page, discussing Laplace's "historical works" (to whose province the *Exposition* belonged):

> *If he writes the history of great astronomical discoveries, he becomes a model of elegance and precision. No leading fact ever escapes him.* […] *Whatever he omits does not deserve to be cited.*

Newton had indeed explained why are the planetary paths eccentric, but did he eliminate chance? No, not at all! Indeed, a similar question remains about the planetary velocities: why are *they* different? I do not know whether this question was formulated earlier.

I have only touched on the general problem of the role of randomness in natural sciences and only allow myself one pertinent reference (out of several possible) to Maxwell:

> *The form and dimension of the orbits of the planets* […] *are not determined by any law of nature, but depend upon a particular collocation of matter. The same is the case with respect to the size of the earth.* Maxwell (1873/1969, p. 360).

I prefer to say: the particular arrangement of matter and velocities in the Solar system.

Acknowledgement. I am grateful to Professor G. Tee (Auckland, N. Z.) for helpful comments.

Notes

1. I have discussed Aristotle earlier (Sheynin 1974, § 2.2) but did not correctly interpret one of his examples. Concerning § 2 below, see also Sheynin (1974, § 8.1.1). I am referring to vol. 2 of Aristotle's *Works* edited by D. Ross (vols 1– 12. Oxford, 1908 – 1954). Below, I refer to another of his contributions from vol. 8 of the same edition. There also exists an edition of Aristotle's *Complete Works* (vols 1 – 2. Princeton, 1984) whose composition is slightly different; the order of the contributions also differs, and the numbering of the pages and lines is therefore different.

2. "Une cause très petite, qui nous échappe, determine un effet considerable […] et alors nous disons que cet effet est dû au hazard" Poincaré (1912/1987, p. 4).

3. I refer to the Russian translation of that source, cf. its bibliographic description.

4. "Nous les attribuons au hazard, parce que leurs causes sont trop compliquées et trop nombreuses […] nous n'avons que de petites causes, mais chacune d'elles ne produit qu'un petit effet; c'est par leur union et par leur nombre que leurs effets deviennent redoutables" (Poincaré 1912/1987, p. 10).

5. "Dans chaque domaine, les lois précises ne décidaient de tout, elles traçaient seulement les limites entre lesquelles il était permis au hazard de se moivoir. Dans cette conception, le mot hazard avait un sens précis, objectif" (Poincaré 1896/1912, p. 1).

6. Ekeland (2006) provides pictures of chaotic *clouds*, of exponentially deviating paths of, for example, a ball on a non-elliptical billiard table. Owing to unavoidable small uncertainty of its initial conditions, the path becomes a cloud which fills a certain region. Chaos certainly is a great extension of "small causes leading to considerable effects" (Poincaré, see § 1). However complicated and protracted is a coin toss, it has a constant number of outcomes whereas chaotic motion implies rapid increase of its instability with time and uncountably infinite positions of its possible path.

7. "Cette inégalité [lunaire] quoique indiquée par les observations, était negligee par le plus nombre des astronomes, parce qu'elle ne paraissait pas résulter de la théorie de la pesanteur universelle. Mais, ayant soumis son existence au Calcul des Probabilités, elle me parut indiqués avec une probabilité si forte, que je crus devoir en rechercher la cause. Laplace" (1812/1886, p. 361).

8. "Was aber ist Zufall? Wahrlich, er ist ein höchst abscheulicher Götze und nichts anderes als eine Beschimpfung des höchsten und allmächtigen Gottes und der höchst vollkommenen Welt, der er schuf" (Kepler 1606/2006, p. 163).

Kepler was neither the first, nor the last to deny randomness. Aristotle stated that "None of the traditional sciences busies itself about the accidental […] but only sophistry" (*Metaphysica* 1064b15). He was wide of the mark: the theory of probability "busies itself" not about the accidental, but about its laws. Then, Laplace stated that chance "has no reality in itself" (n'a aucune réalité en lui-même), it only signified our ignorance (Laplace 1776/1891, p. 145). And Darwin thought that variations in his theory were not at all "due to chance", that such an expression only acknowledged "our ignorance" of the proper causes (Darwin 1859/1964, Chapt. 5, p. 131).

9. "Ein Astrologus, der nur den Himmel sihet und von […] zwischenursachen nicht weiss, nur allein probabiliter […] das ist, ein klein wenig mehr dann nichts" […] Kepler (1610/1941, p. 217).

10. "Die Vielheit der Umstände, die an jeglicher Naturbeschaffenheit Anteil nehmen, eine abgemessene Regelmäßigkeit nicht verstattet" (Kant 1755/1910, 1. Hauptstück, p. 269).

11. "Woher sind ihre Umläufe nicht vollkommen zirkelrund? […] Ist es nicht klar einzusehen, dass diejenige Ursache welche die Laufbahne der Himmelkörper gestellet hat, […] es nicht völlig hat ausrichten können […]. Ist nicht das gewöhnliche Verfahren der Natur hieran zu erkennen, welches durch die Dazwischenkunst der verschiedenen Mitwirkungen allemal von der ganz abgemessenen Bestimmung abweichend gemacht wird?" (Kant 1755/1910, 8. Hauptstück, p. 337).

12. "Si le système solaire s'était formé avec une parfaite régularité, les orbites des corps qui le composent seraient des cercles, dont les plans, ainsi que ceux des divers équateurs et des anneaux, coïncideraient avec le plan de l'équateur solaire. Mais on conçoit que les variétés sans nombre qui ont dû exister dans la temperature et la densité des diverses parties de ces grandes masses ont produit les excentricités de leurs orbites, et les déviations de leurs mouvements du plan de cet équateur" (Laplace 1835/1884, Note 7, p. 504).

References

Boscovich, R. (1758, in Latin/1966), *Theory of Natural Philosophy*. Cambridge, Mass.

Cournot, A. A. (1843), *Exposition de la théorie des chances et des probabilités*. Paris, 1984.

Darwin, C. (1859/1964), *Origin of Species*. Cambridge, Mass.

De Moivre, A. (1718/1967), *Doctrine of Chances*. New York. This is a reprint of the third edition of 1756.

Ekeland, I. (2006), *The Best of All Possible Worlds.* Chicago and London.

Fourier, J. B. J. (1829), Historical Eloge of the Marquis De Laplace. *London, Edinb. and Dublin Phil. Mag*., ser. 2, vol. 6, pp. 370 – 381. The original French text was only published in 1831.

Great Books (1952), *Great Books of the Western World*, vols 1 – 54. Chicago.

Harvey, W. (1651 in Latin/1952), *Anatomical Exercises in the Generation of Animals*. In *Great Books* (1952, vol. 28, pp. 329 – 498).

Kant, I. (1755/1910), *Allgemeine Naturgeschichte und Theorie des Himmels*. In author's *Ges. Schriften*, Abt. 1, Bd. 1. Berlin, pp. 215 – 368.

Kepler, J. (1596/1963), *Mysterium Cosmographicum. Ges. Werke*, Bd. 8. München, pp. 7 – 128. The 1963 version is a reprint of the second edition of 1621 with additions having been inserted by Kepler to the first edition to many chapters. German translation: Augsburg, 1923; München – Berlin, 1936.

--- (1606, in Latin/2006), *Über den Neuen Stern im Fuß des Schlangenträger*. Würzburg.

--- (1609, in Latin/1992), *New Astronomy*. Cambridge.

--- (1610/1941), Tertius interveniens. In author's *Ges. Werke*, Bd. 4. München, pp. 149 – 258. In German.

--- (1619/1997), *Harmony of the World*. Philadelphia, Book 5 (in Latin, as was the entire contribution). German translation: Münich – Berlin, 1939.

--- (1620 – 1621/1952), *Epitome of Copernican Astronomy*, books 4 – 5. In *Great Books* (1952, vol. 16, pp. 845 – 1004). The entire contribution was initially in Latin.

Lamarck J. B. (an 8, 1800), *Annuaire météorologique*, t. 1. Paris.

--- (1810 – 1814, manuscript), *Aperçu analytique des connaissances humaines*. Partly published: Vachon, M., et al (1972), *Inédits de Lamarck*. Paris, pp. 69 – 141. Russian translation of entire work in author's *Izbrannye Proizvedenia* (Sel. Works), vol. 2. Moscow, 1959, pp. 93 – 662.

--- (1815), *Histoire naturelle des animaux sans vertèbres*, t. 1. Paris.

Laplace P. S. (1776/1891), Recherches sur l'intégration des equations différentielles. *Oeuvr. Compl.*, t. 8. Paris, pp. 69 – 197.

--- (1796, 1798/1799, 1808, 1813, 1835/1884), *Exposition du système du monde*. *Oeuvr. Compl.*, t. 6. Paris.

--- (1812/1886), *Théorie analytique des probabilités. Oeuvr. Compl.*, t. 7. Paris.

--- (1814, in French/1995), *Philosophical Essay on Probabilities*. New York.

Maupertuis, P. L. M. (1756), Sur le divination. *Oeuvres*, t. 2. Lyon, 1756, pp. 298 – 306.

Maxwell, J. C. (1873, manuscript; publ. 1882/1969), Discourse on molecules. In Campbell, L., Garnett, W. (1882/1969), *Life of Maxwell*. New York – London, pp. 358 – 361.

Newton, I. (1704/1931), *Opticks.* London, 1931. *Queries* were added later, from 1717 onward, and the edition of 1931 (reprinted in 1952) was based on that of 1730.

Poincaré, H. (1896), *Calcul des probabilités*. Paris. Second edition, 1912, reprint Sceaux, 1987.

Sheynin, O. (1974), On the prehistory of the theory of probability. *Arch. Hist. Ex. Sci.*, vol. 12, pp. 97 – 141.

---(1991), The notion of randomness from Aristotle to Poincaré. *Math., Inform., Sci. Hum.*, 29^{e} année, No. 114, pp. 41 – 55. Also in my *Russian Papers on the History of Probability and Statistics.* Berlin, 2004 and at www.sheynin.de