CLARIFICATIONS AND IMPROVEMENTS
Probability and Finance: It's Only a Game!

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This document lists some clarifications that the authors may add if the book has a second edition. Some inconsistencies and possible improvements are also noted.

p. 5, line 11 from bottom: In some contexts, it may be more appropriate to say that Forecaster's strategy is rejected. See p. 58 and §8.2.

p. 18, line 7 of first full paragraph: In addition to subsequences selected in advance, von Mises also considered subsequences selected in such a way that the inclusion of the nth trial depends on the outcomes of preceding trials but not on the outcome of the nth trial itself.

p. 49, first line: The phrase "classical probability theory" is used here in a rather wide sense.

p. 56, last two paragraphs: Perhaps it is not so helpful to emphasize that the hypothesis of the impossibility of a gambling system can be stated *before* one computes probabilities for events in the game. The important points are (1) the hypothesis is not a mathematical fact about the probabilities in the game, and (2) although it provides a way of interpreting small probabilities, it can be expressed without using the concept of probability.

p. 58: Dawid's ideas are also discussed on p. 8. The interpretation of a probability distribution as a strategy for Forecaster is discussed in more depth in §8.2.

p. 64, last full paragraph: The significance of forbidding or limiting Skeptic's borrowing is also discussed on p. 17.

p. 117, first line of Convergence Check: "step" $\mapsto$ "round"

p. 130, second line of (6.15): "$\partial D \partial s$" $\mapsto$ "$\partial s \partial D$"

p. 134, line before (6.23), p. 139, second line of Approximation Theorem, p. 139, second line of Lemma 6.2, and p. 141, line 4: "increasing" here is meant in the wide sense of "nondecreasing" (we are not always consistent: e.g., line 5 from the bottom of p. 139 and line 4 of p. 141 have "nondecreasing").

p. 160, last paragraph before Example 2: more accurately, the individual variables should not contribute too substantially to the total variance or should themselves be almost Gaussian.

p. 169, fourth full paragraph, last sentence: it would be more natural to allow τ to take value 0 as well.

p. 171, first line of footnote 3: "this mathematical observation" $\mapsto$ "the picture"

pp. 173–175: For consistency with earlier notation, ω should be replaced with ξ throughout the subsection entitled **The Game-Theoretic Form of Lindeberg's Theorem with Measurability**. There are two occurrences on p. 173 and ten on p. 174.

pp. 189–191: In the subsection entitled **Quantum Mechanics**, the eigenvalues are written as $\alpha_1, \alpha_2, \ldots$ and sometimes as $a_1, a_2, \ldots$. All the a should be changed to α. This involves 2 changes on p. 189, 4 changes on p. 190, and 5 changes on p. 191.

p. 202, lines 4 and 5 from bottom: "how to replace it with a more practical instrument." $\mapsto$ "how it might be replaced with a more ordinary derivative."

p. 212, line 7: Perhaps a reference (such as [262], Volume 1, Corollary 25.6 on p. 61) should be provided concerning the continuity of sample paths.

p. 218, penultimate line: Other, hardly less important, motivations are that σ is not known in advance and the whole model (9.12) is empirically shaky.

p. 280, line 5; p. 281, line 5; p. 300, line 3 above Theorem 12.1: Remove "$\inf_n S_n$ is positive and not infinitesimal,".

p. 302, third line above the itemized list: calls and puts will definitely be too awkward if only one call or put is used in hedging: because of the volatility smile, the implied price $D(t)$ (and so the implied price of $U(S(T))$ as well) will depend on the strike, which means that the hedging error will be high at least for some strikes.

p. 304, second line above the protocol: "prices" $\mapsto$ "upper and lower prices"

p. 310, bottom line: Remove "$\inf_n S_n^{(0)}$ is positive and not infinitesimal,".

p. 324, penultimate and bottom lines: "If an investor intends to use outside information that he acquires after the game begins" $\mapsto$ "If an investor intends to use outside information that he acquires after the game begins or, what is even more important (since outside information can be included in $\mathbf{W}$), plans to develop his strategy in the process of the game"

p. 327, second line after the protocol: this remark is applicable to any two instruments G and H, maybe both active.

p. 335, second line from bottom: this sense of "binding" (Reality will respect Forecaster's probabilities on average) is only one of many implications of the fundamental interpretative hypothesis

p. 339, first two lines of the proof: our wording is sloppy here: since S is not given at this point, the assumption $\mathbf{var}_S(2) < \infty$ does not make sense at this point. An analogous remark can be made about **p. 340, first line**.

p. 359, end of first paragraph: "can be used to model a sequence of horse races [61, 169]." $\mapsto$ "can model horse races (Kelley 1956, Cover and Ordentlich 1996)."

p. 362, second line of the second paragraph of subsection "Deducing the Martingale Strong Law of Large Numbers": "$(\Omega, \mathcal{F}, P)$" $\mapsto$ "$(\Omega, \mathcal{F}, \mathbb{P})$"; in **Equation (15.15)**, $\mathbb{E}$ stands for the expectation w.r. to $\mathbb{P}$.

p. 364, second line from bottom: "trial" $\mapsto$ "round"

p. 365, second line of the second full paragraph: "do better" $\mapsto$ "do better (or do worse)"

p. 366, first line after Equation (15.21): the C introduced in this line is different from the C in (15.21); both are used afterwards, but it is always easy to tell which is which

p. 368, lines 3 and 4: these expressions for $\mathcal{I}_n$ and $\mathcal{K}_n$ assume zero interest rates, which imposes restrictions on the *numéraire* (it should be tradable, such as a bond or a market index).

pp. 367–371: For a deeper treatment of the relation between risk and return in the game-theoretic finance, see Game-Theoretic Probability Project Working Papers 1 and 2 (can be downloaded from `http://www.cs.rhul.ac.uk/~vovk/book/`).

p. 374, last paragraph: both "options markets" and "option markets" are used (see the first and penultimate lines)

p. 379, citation [60]: "Jean-Michel Courtault et al. Louis Bachelier: On the centenary of Théorie de la Speculation." $\mapsto$ "Jean-Michel Courtault, Yuri Kabanov, Bernard Bru, Pierre Crépel, Isabelle Lebon, and Arnaud Le Marchard. Louis Bachelier on the centenary of *Théorie de la Spéculation*."

p. 380, citation [77]: "K. E. Dambis" $\mapsto$ "Karl E. Dambis"

p. 395, citations [330] and [331]: The updated citations are

330. Vladimir G. Vovk. Probability theory for the Brier game. *Theoretical Computer Science*, 261:57–79, 2001.

331. Vladimir G. Vovk. Kolmogorov's complexity conception of probability. In Vincent F. Hendricks, Stig Andur Pedersen, and Klaus Frovin Jørgensen, editors, *Probability Theory: Philosophy, Recent History and Relations to Science*, volume 297 of *Synthese Library*, pp. 51–69. Kluwer, Dordrecht, 2001.

p. 396, citation [338], line 5: we could have spelled "Doeblin" more consistently on pp. 198, 396, and 407.

p. 403: After line 4 (log), add: "ln: logarithm to the base e, 37"

p. 405: After line 2 (σ-algebra), add: "$\sqrt{dt}$-effect, 201, 205"

p. 406: "Britten-Jones, Mark" $\mapsto$ "Britten-Jones, Mark (born 1963)"; "Calvet, Laurent" $\mapsto$ "Calvet, Laurent (born 1969)"

p. 407: Merge the entry "Cournot's bridge" with the subentry "Cournot's bridge" of the entry "Cournot, Antoine Augustin (1801–1877)"

p. 407: Add "Cover, Thomas M. (born 1938), 359". "Davis, Morton" ↦ "Davis, Morton (born 1930)"; "Emanuel, David" ↦ "Emanuel, David (born 1949)"

p. 408: The entry for Gnedenko can be extended to "Gnedenko, Boris V. (1912–1995)".

p. 409: Add "Kelley Jr., John L. (1916–1999), 359".

p. 409: Add subsubentry "fuzzy, 362" to the subentry "strong" of the entry "Law of large numbers"

p. 410, entry for "Lyapunov": we usually spell Lyapunov's first name as "Aleksandr" (transliteration from Russian).

p. 410: The first four entries in the second column should be made subentries under "Market". "Mercurio, Fabio" ↦ "Mercurio, Fabio (born 1966)"

p. 410, entry for "Martin": it is awkward that the entry for "Martin" is so far after the entry for "Martin's theorem"

p. 411: Add "Ordentlich, Erik, 359".

p. 412: Under "Protocol", there should be subentries for "coherent" and "terminating". "Schwarz, Gideon" ↦ "Schwarz, Gideon (born 1933)"

p. 414: The entry "Variation spectrum" should be merged with the corresponding subentry under "Variation". "Walley, Peter" ↦ "Walley, Peter (born 1953)"